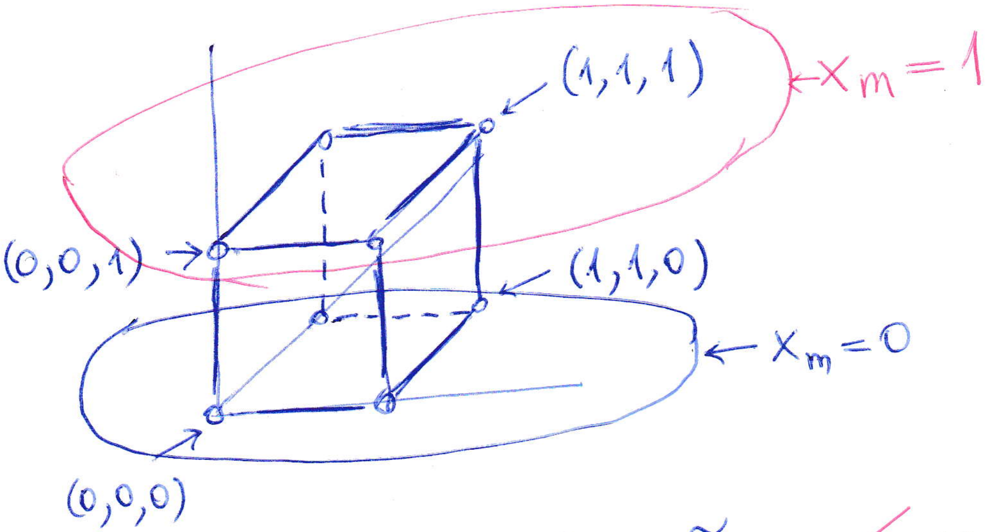
Binary REED-MULLER codes

Hypercube: $m=2$: $(0,0), (0,1), (1,0), (1,1)$

$\{0,1\}^m = GF(2)^m$ $\quad$ $m=3$: $(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)$

lexicographic ordering



$$f = x_1^3 x_2 + x_3^2 + x_1 x_2 + x_1^2 + x_2 x_3 \rightsquigarrow \tilde{f} = \cancel{x_1 x_2} + x_3 + \cancel{x_1 x_2} + x_1 + x_2 x_3$$

$$= x_3 + x_1 + x_2 x_3$$

multilinear

$f(x_1, \dots x_m) \in GF(2)[x_1, \dots x_m]$

Evaluation

$$V(f) = (f(0, \dots 0), f(0, \dots 0, 1, 0), \dots, f(1,1,1 \dots 1))$$

for the f above ($m=3$): $(0, 1, 0, 0, 1, 0, 1, 1)$

$$w(V(f)) = w(V(f|_{x_m=0})) + w(V(f|_{x_m=1}))$$

$\#(x_1, \dots x_m)$: $f(x_1, \dots x_m) = 1$

$\boxed{\#(x_1, \dots x_{m-1}) : f(x_1, \dots x_{m-1}, 0) = 1}$

$\boxed{\#(x_1, \dots x_{m-1}) : f(x_1, \dots x_{m-1}, 1) = 1.}$