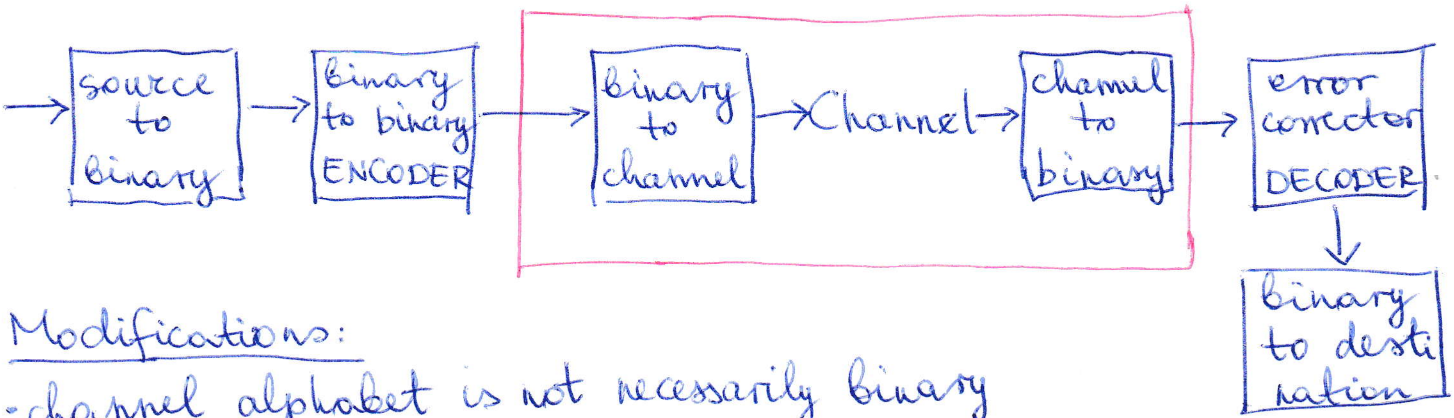
Error-corr. codes

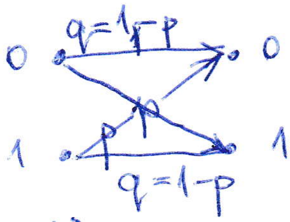


Modifications:
- channel alphabet is not necessarily binary
- sometimes error detection is enough

Channel: discrete memoryless channel (DMC)

$P(b_j | a_i)$ = prob. b_j is received if a_i is transmitted

BSC (binary symm. channel)



(More general: q-ary symm. channel)

Example (toss a coin twice) (H,T) Channel p=0.02 (2%)

Encoding: HH → 0000, TH → 1001, HT → 0111, TT → 1110

Decoding: if something different is received, assume that one of the first 3 bits is in error (∃! good msg)

Information rate $\frac{2}{4} = 0.5$ (originally "2 bits", we use "4 bits")

Prob. of error 1.12% (0.0112)

General decoding ideas:
Maximum likelihood
Maximum a posteriori
Simplified: assume that the least number of errors occured
(toss a coin three times)

$a_1, a_2, a_3, a_4 = a_2 + a_3, a_5 = a_3 + a_1, a_6 = a_1 + a_2$ ← inf. rate $\frac{1}{2}$

Prob. of error ≈ 0.29%

$\underbrace{H,H,T,H,}$ T, T, T, H, T, H, H, H, H, T, H ...

divide it in length (k) parts ⇝ extend this part to $\boxed{n}$ bits ← inf. rate $\frac{k}{n} = R$ Prob. error < ε? (for appropriate k, n and fixed R)

Block codes